Appendix H: Mathematical Structure of the Ukraine Model

This document presents the algebraic formulation of a general-equilibrium numeric-simulation model of the Ukrainian economy. This model largely follows the structure of a simulation model of the Belarus economy (Balistreri et al., 2017).

The model includes 45 goods and services, which are purchased by households, firms, and the government. Let the goods and services be indexed by $g \in G$. Divide these goods and services into the following three categories that define their treatment in the model formulation: (a.) Business Services, characterized by monopolistic competition and foreign direct investment (FDI), indexed by $i \in I \subset G$; (b.) Dixit-Stiglitz manufacturing sectors, characterized by monopolistic competition, indexed by $j \in J \subset G$; and (c.) Constant Returns To Scale (CRTS) goods indexed by $k \in K \subset G$. In the current aggregation there are 9 elements in I, 13 elements in J, and 23 elements in K. Goods and services are also classified by their associated region, indexed by $r \in R$, where there are 8 regions: Ukraine (or the domestic region), Turkey, the European Union, the Russian Federation, the United States, China, the group of countries with which Ukraine has an FTA (other than the EU), and the rest-of-world region. The accounts track the incomes of the representative household in Ukraine decomposed by the mobile factors of production (unskilled labor, skilled labor, capital) and sector-specific factors (resource/land input and capital for the increasing returns sectors).

Table 1 summarizes the equilibrium conditions and associated variables. The non-linear system is formulated in GAMS/MPSGE and solved using the PATH algorithm. We proceed with a description and algebraic representation of each of the conditions

Table 1: General equilibrium conditions

Equilibrium Condition	Equation		Associated Variable
Dual representation of preferences and technologies:			
Armington unit-cost functions	(1)	$\forall i \in I$	A^g : Armington Activity
	(2)	$\forall j \in J$	
	(3)	$\forall k \in K$	
Dixit-Stiglitz price indexes	(4)	$\forall g \in (I \cup J)$	Q_r^g : D-S Activity by region
Zero Profits for Dixit-Stiglitz firms	(5)	$\forall g \in (I \cup J)$	N_r^g : Number of Firms
Dixit-Stiglitz composite input prices	(6)	$\forall g \in (I \cup J)$ and $r = D$	Z_r^g : IRTS resource use
	(7)	$\forall j \in J$ and $r \neq D$	
	(8)	$\forall i \in I$ and $r \neq D$	
Input-output technologies	(10)	$\forall g \in G$	Y^g: Production level
Constant elasticity of transformation	(11)	$\forall g \in G$	X^g: Index on CET activity
Perfectly elastic exports	(12)	$r \neq D$	EX_r^g: Exports
Iso-elastic exports	(13)	$r \neq D$	EX_r^g: Exports
Imports	(14)	$\forall g \in G$ and $r \neq D$	IM_r^g: Imports (net of FDI-firm imports)
Unit expenditure function	(15)		U: Household utility index
Unit cost of public purchase	(16)		PUB: Government Activity
Unit cost of investment	(17)		INV: Investment Activity
Market clearance conditions:			
Composite goods and services	(18)	$\forall g \in G$	PA^g : Composite price indexes
D-S composites	(21)	$\forall g \in (I \cup J)$ and $r \neq D$	P_r^g: Prices of D-S composites
	(22)	$\forall g \in (I \cup J)$ and $r = D$	
Markets for IRTS composite input	(23)	$\forall g \in (I \cup J)$	PMC^g: Composite input prices
Markets for domestic output	(24)	$\forall k \in K$	PD^g: Domestic output prices
	(25)	$\forall i \in I$	
	(26)	$\forall j \in J$	
Markets for export output	(27)	$\forall g \in G$ and $r \neq D$	PX_r^g: Export output prices
Markets for imports	(28)	$\forall i \in I$ and $r \neq D$	PM_r^g: Import prices
	(29)	$\forall j \in J$ and $r \neq D$	
	(30)	$\forall k \in K$ and $r \neq D$	
Factor markets	(31)	$\forall f \in F$	PF_f: Factor prices
Resource and land factor	(32)		PF_{res}^g: Factor prices
IRTS specific factors	(33)	$\forall g \in (I \cup J)$	PZ_r^g: Sector-specific capital price
Fixed real investment	(34)		$PINV$: Unit cost of investment
Fixed real public spending	(35)		PG: Unit cost of public good
Nominal utility equals Income	(36)		PC: Unit expenditure index
Balance of payments	(37)		PFX: Price of foreign exchange
Income balance:			
Domestic agent income	(38)		RA_h: Household Income
Government budget	(39)		$GOVT$: Government spending
Foreign Entrepreneur	(40)		FE: External agent income
Auxiliary Conditions:			
Fixed real public spending	(41)		T: Index on direct taxes

itemized in Table 1.

1 Dual representation of technologies and preferences

Technologies and preferences are represented through value functions that embed the optimizing behavior of agents. Generally, any linearly-homogeneous transformation of inputs into outputs is fully characterized by a unit-cost (or expenditure) function. Setting the output price equal to optimized unit cost yields the equilibrium condition for the activity level of the transformation. That is, a competitive constant-returns activity will increase up to the point that marginal benefit (unit revenue) equals marginal cost. In the case of the Ukraine model not all transformations are constant returns, so there are exceptions. In general, however, we will use the convention of setting unit revenues (left-hand side) equal to unit cost (right-hand side) and associating this equilibrium condition with a transformation activity level.

Agents in Ukraine wishing to purchase a particular good or service g face an aggregate price PA^g. In constructing the aggregate prices, we will rely on the following notation for the component prices:

PD^g Price of domestic output ($\forall g \in G$),

PM_r^g Price of cross-border imports from region r of Business Services and CRTS goods ($\forall g \in (I \cup K)$),

P_r^g Dixit-Stiglitz price index on region-r varieties ($\forall g \in (I \cup J)$).

Assuming a Constant Elasticity of Substitution (CES) aggregation of the components we equate the prices to the CES unit-cost functions:

$$PA^i = \left(\sum_r \phi_r^i (P_r^i)^{1-\sigma_F^i} + \sum_r \phi_r^{i,m} [(1 + t_i^{m,int}) PM_r^i]^{1-\sigma_F^i} \right)^{1/(1-\sigma_F^i)} \quad (1)$$

$$PA^j = \left(\sum_r \phi_r^j (P_r^j)^{1-\sigma_F^j} \right)^{1/(1-\sigma_F^j)} \quad (2)$$

$$PA^k = \left(\phi_D^k [(1 + t_k^{d,int}) PD^k]^{1-\sigma_{DM}^k} + \sum_r \phi_r^k [(1 + t_k^{m,int}) PM_r^k]^{1-\sigma_{DM}^k} \right)^{1/(1-\sigma_{DM}^k)}, \quad (3)$$

where $\sigma_F^g \forall g \in (I \cup J)$ is the Dixit-Stiglitz elasticity of substitution and σ_{DM}^k is the Armington elasticity of substitution on CRTS goods. $t_g^{d,int}$ is the domestic intermediate input tax on purchases of good g, whereas $t_g^{m,int}$ is the imported intermediate input tax on purchases of good g. While we keep track of VAT, excise tax, and duty separately in the simulation model, to simplify the equations in this Appendix, $t_g^{d,int}$ and $t_g^{m,int}$ are considered as the aggregated tax rates. For the Dixit-Stiglitz price index, the intermediate input taxes are already included as shown below. The ϕ parameters are CES distribution parameters that indicate scale and weighting of the arguments. These are calibrated to the Ukrainian social accounts such that the accounts are replicated in the benchmark equilibrium.

For the IRTS sectors we have the Dixit-Stiglitz price indexes. These are functions of the number of varieties, firm-level costs, and the optimal markup. Assuming each firm is small relative to the size of the market the demand elasticity for a firm's variety is σ_F^g and the optimal markup over marginal cost is given by $1/(1-\frac{1}{\sigma_F^g})$. Let marginal cost equal $PMC_r^g \; \forall g \in (I \cup J)$, which is the price of a composite input to the Dixit-Stiglitz firms associated with region-r, and let the number of varieties by region equal $N_r^g \; \forall g \in (I \cup J)$. The price indexes for the Dixit-Stiglitz goods are thus given by

$$P_r^g \;=\; \left[N_r^g \left(\frac{PMC_r^g}{1-\frac{1}{\sigma_F^g}} \right)^{1-\sigma_F^g} \right]^{1/(1-\sigma_F^g)} \qquad \forall g \in (I \cup J). \tag{4}$$

In equilibrium, the number of varieties by region adjusts such that we have zero profits. Denote the Dixit-Stiglitz composite activity level associated with equation (4) by $Q_r^g \; \forall g \in (I \cup J)$. Given the Dixit-Stiglitz aggregation of varieties each firm produces a quantity $Q_r^g (N_r^g)^{\sigma_F^g/(1-\sigma_F^g)}$. Assuming that fixed and variable costs are satisfied using the same input technology, and a firm-level fixed cost of f_r^g (in composite input units), we have the zero profit condition

$$f_r^g - \frac{Q_r^g (N_r^g)^{\sigma_F^g/(1-\sigma_F^g)}}{\sigma_F^g - 1} = 0 \quad \forall g \in (I \cup J). \tag{5}$$

The technologies for producing the composite inputs for use in the Dixit-Stiglitz sectors depend on the type of sector. For all of the sectors there is a sector-specific capital input from the respective source region. Let $PZ_r^g\ \forall g \in (I \cup J)$ be the price of this sector-specific capital input. Domestic firms (producing goods or services) use domestic inputs, so the unit cost function is given by

$$PMC_r^g = \left[\theta_{Zr}^g (PZ_r^g)^{1-\epsilon_r^g} + \theta_{Dr}^g [(1+t_g^{d,int})PD^g]^{1-\epsilon_r^g}\right]^{1/(1-\epsilon_r^g)}, \quad \text{for} \quad r = D; \tag{6}$$

where ϵ_r^g is the elasticity of substitution between the sector-specific capital input and other inputs, and the θ's are the CES distribution parameters. Imports of Dixit-Stiglitz goods embody the gross of tariff imported inputs:

$$PMC_r^j = \left[\theta_{Zr}^j (PZ_r^j)^{1-\epsilon_r^j} + \theta_{Mr}^j [(1+t_j^{m,int})PM_r^j]^{1-\epsilon_r^j}\right]^{1/(1-\epsilon_r^j)}, \quad \text{for} \quad r \neq D. \tag{7}$$

FDI firms, on the other hand, use domestic inputs as well as a specialized imported service from the sources region. The price of the specialized imports equals the price of foreign exchange (denoted PFX) times one plus the tariff rate (denoted $t_i^{m,int}$). The unit cost for FDI firms is thus given by the following:

$$PMC_r^i = \left[\theta_{Zr}^i (PZ_r^i)^{1-\epsilon_r^i} + \theta_{Dr}^i [(1+t_i^{d,int})PD^i]^{1-\epsilon_r^i} + \theta_{Mr}^i [(1+t_i^{m,int})PM_r^i]^{1-\epsilon_r^i}\right]^{1/(1-\epsilon_r^i)},$$
$$\text{for} \quad r \neq D. \tag{8}$$

For the CRTS sectors and upstream of the IRTS technologies, we have domestic production in accordance with the input output data. Denote the price of this output PY^s, for $s \in G$. The technology includes an upstream Cobb-Douglas value-added nest which then combines business services and ultimately then this composite combines with other intermediates in fixed proportions. Let PF_f indicate the price of mobile factor of production and let PF_{res}^g be the price of resource/land factor in sector g. P_s^{vas} is the value-added business-services composite price for sector s. The composite of business services and value added is the CES aggregate of two Cobb-Douglas aggregates as follows:

$$P_s^{vas} = \left[\left(\prod_g [\theta_g^s PA^g + \theta_{mg}^{s,g} PA^{mg}]^{\alpha_g^s}\right)^{1-\sigma_{vas}} + \left(\prod_f [(1+t_{fs})PF_f]^{\alpha_f^s} [(1+t_s^{res})PF_{res}^g]^{\alpha_{res}^s}\right)^{1-\sigma_{vas}}\right]^{1/(1-\sigma_{vas})} \quad (9)$$

where t_{fs} and t_s^{res} are the factor taxes. Note that we have five transport margin goods (mg), and they are combined with corresponding intermediate input in a fixed proportion (or Leontief function). The substitution elasticity between value added and the business services composite is given by σ_{vas}. With P_s^{vas} established, the top-level CES unit cost function for sector s is given by

$$PY^s = \left(\beta_{vas}^s (P_s^{vas})^{1-\sigma_s} + \sum_{g \neq I} \beta_g^s (\theta_g^s PA^g + \theta_{mg}^{s,g} PA^{mg})^{1-\sigma_s}\right)^{1/(1-\sigma_s)}, \quad (10)$$

where the α and β are share and scale parameters determined in the calibration to the input-output accounts and σ_s is the substitution elasticity at the top-level.

A constant elasticity of transformation (CET) activity splits domestic output (with a unit value PY^k) into goods destine for domestic versus the region-specific export markets. Let the export price (for goods destine for region $r \neq D$) be PX_r^g then the CET technology is given by

$$\left[\gamma_D^g (PD^g)^{1+\sigma_\tau} + \sum_{r \neq D} \gamma_r^g (PX_r^g)^{1+\sigma_\tau}\right]^{1/(1+\sigma_\tau)} = PY^g, \quad (11)$$

where σ_τ indicates the elasticity of transformation and the γ are the CET distribution parameters.

For most sectors, export demand is perfectly elastic, and the export commodity is traded for foreign exchange at a fixed rate. Let PFX equal the price of foreign exchange, and with a choice of units such that all gross of tax unit export prices are one at the benchmark, we have the following specification for the export activities:

$$PFX = (1 + t_g^{exp}) PX_r^g \quad \text{for} \quad r \neq D, \tag{12}$$

where t_g^{exp} is the export tax. For some sectors where it is reasonable to consider some limitation of export demand response in our trade agreement scenarios, we consider iso-elastic export demand function as follows:

$$EX_r^g = \left[\frac{(1 + t_k^{exp}) PX_r^g}{PFX} \right]^{-\sigma_F^g} \quad r \neq D. \tag{13}$$

Cross-border imports are purchased at the price of foreign exchange, which sets up the arbitrage condition for each import activity;

$$PM_r^g = PFX \quad \text{for} \quad r \neq D. \tag{14}$$

Final demand includes three categories: household demand, government demand, and investment. The representative agents for each household h are assumed to have identical Cobb-Douglas preferences over the aggregated goods and services. The preferences are specified via a unit expenditure function associated with an economy-wide utility index (U). Let PC be the true-cost-of-living index indicated by the following unit expenditure function:

$$PC = \prod_g [(1 + t_g^{cons}) PA^g]^{\mu_C^g}, \tag{15}$$

where the μ are value shares. The government faces a Leontief price index, PG, for government purchases:

$$PG = \sum_g \mu_G^g (1 + t_g^{gov}) PA^g. \tag{16}$$

Similarly the price of investment, $PINV$ is a Leontief aggregation of commodity purchases:

$$PINV = \sum_g \mu_{INV}^g (1 + t_g^{inv}) PA^g. \tag{17}$$

Equations (1) through (17) define all of the transformation technologies for the model. Next we turn to a specification of the market clearance conditions for each price.

2 Market clearance conditions

For each good or service there is a market, and, for any non-zero equilibrium price, supply will equal demand. We will use the convention of equating supply, on the left-hand side, to demand, on the right-hand side. The unit-value functions presented above are quite useful in deriving the appropriate compensated demand functions, by the envelope theorem (Shephard's Lemma).

Supply of the composite goods and services, trading at PA^g, is given by the activity level, A^g, and demand is derived from each production or final demand activity that uses the good or service. The market clearance condition is given by

$$A^g = \sum_s h_{gs}(Y^s, \boldsymbol{p}) + \mu_C^g U \frac{PC}{(1 + t_g^{cons}) PA^g} + \mu_G^g PUB + \mu_{INV}^g INV, \tag{18}$$

where $h_{gs}(Y^s, \boldsymbol{p})$ are the conditional input demands (as a function of output and the price vector. These are found by taking the partial derivative of the unit cost function for sector s with respect to the gross of tax price of input g, but they are differentiated for business service goods and others as the positions in nesting structures differ. Input demand of non-business service goods are following:

$$h_{gs}(Y^s, \boldsymbol{p}) = \theta_g^s \beta_g^s Y^s \left(\frac{PY^s}{PA_g} \right)^{\sigma_{vas}} \quad \forall g \in (J \cup K). \tag{19}$$

Input demand for business service goods are following:

$$h_{is}(Y^s, \boldsymbol{p}) \;\; = \;\; \theta_i^s \alpha_i^s \beta_{vas}^s Y^s \left(\frac{P_s^{srv}}{PA_i}\right)\left(\frac{P_s^{vas}}{P_s^{srv}}\right)^{\sigma_{vas}}\left(\frac{PY^s}{P_s^{vas}}\right)^{\sigma_s} \tag{20}$$

where P_s^{srv} is the composite price of business services inputs: $P_s^{srv} = \prod_i [\theta_i^s PA^i + \theta_{mg}^{s,g} PA^{mg}]^{\alpha_i^s}$. Note that for the transportation margin sectors, we have similar input demand of margin input, and thus we have intermediate input demand and margin input demand.

For the IRTS sectors we have market clearance for the Dixit-Stiglitz regional composites:

$$Q_r^g \;\; = \;\; A^g \left(\frac{PA^g}{P_r^g}\right)^{\sigma_F^g} \forall g \in (I \cup J), \;\; \text{for} \;\;\; r \neq D; \tag{21}$$

and for domestic firms we include demand for the Dixit-Stiglitz exports

$$Q_D^g \;\; = \;\; A^g \left(\frac{PA^g}{P_D^g}\right)^{\sigma_F^g} + \sum_r EX_r^g \quad \forall g \in (I \cup J). \tag{22}$$

The IRTS composite input (trading at PMC_r^g) is supplied by an activity, denoted Z_r^g $\forall g \in (I \cup J)$, and is demanded by the firms:

$$Z_r^g \;\; = \;\; f_r^g N_r^g + Q_r^g (N_r^g)^{1/(1-\sigma_F^g)} \quad \forall g \in (I \cup J). \tag{23}$$

To derive (23) recall that firm-level output is $Q_r^g (N_r^g)^{\sigma_F^g/(1-\sigma_F^g)}$ so the use of the input across all firms is $Q_r^g (N_r^g)^{1/(1-\sigma_F^g)}$ plus the total input use on fixed costs, $f_r^g N_r^g$.

Market clearance for the domestic output of CRTS sectors depends on supply from the CET activity and demand from the Armington activity:

$$\gamma_D^k X^k \left(\frac{PD^k}{PY^k}\right)^{\sigma_\tau} \;\; = \;\; \phi_D^k A^k \left(\frac{PA^k}{PD^k}\right)^{\sigma_{DM}^k}. \tag{24}$$

For IRTS sectors, supply is given by the CET activity. Output is then demanded by either the domestic or FDI firms. The market clearance conditions are given by

$$\gamma_D^i X^i \left(\frac{PD^i}{PY^i}\right)^{\sigma_\tau} = \theta_{DD}^i Z_D^i \left(\frac{PMC_D^i}{PD^i}\right)^{\epsilon_D^i} + \sum_{r \neq D} \theta_{Dr}^i Z_r^i \left(\frac{PMC_r^i}{PD^i}\right)^{\epsilon_r^i} \tag{25}$$

for the service sectors, and

$$\gamma_D^j X^j \left(\frac{PD^j}{PY^j}\right)^{\sigma_\tau} = \theta_{DD}^j Z_D^j \left(\frac{PMC_D^j}{PD^j}\right)^{\epsilon_D^j} \tag{26}$$

for the Dixit-Stiglitz goods sectors.

Market clearance for exports is given by the CET supply function and demand is given by the export activity level.

$$\gamma_r^g X^g \left(\frac{PX_r^g}{PY^g}\right)^{\sigma_\tau} = EX_r^g, \quad \text{for} \quad r \neq D. \tag{27}$$

Import supply is perfectly elastic and import demand is derived from the Armington activities or embodied in the foreign Dixit-Stiglitz firm's inputs. For $r \neq D$, we have the following:

$$IM_r^i = \phi_r^i A^i \left(\frac{PA^i}{PM_r^i}\right)^{\sigma_F^i} \tag{28}$$

$$IM_r^j = \theta_{Mr}^j Z_r^j \left(\frac{PMC_r^j}{PM_r^j}\right)^{\epsilon_r^j} \tag{29}$$

$$IM_r^k = \phi_r^k A^k \left(\frac{PA^k}{PM_r^k}\right)^{\sigma_{DM}^k}. \tag{30}$$

Factor markets clear, where factor supply is given by the exogenous endowments to households, denoted $\overline{S}_f$, and input demands are derived from the cost functions:

$$\overline{S}_f = \sum_s \alpha^s_f \beta^s_{vas} Y^s \left(\frac{P^{va}_s}{(1+t_{fs})PF_f} \right) \left(\frac{P^{vas}_s}{P^{va}_s} \right)^{\sigma_{vas}} \left(\frac{PY^s}{P^{vas}_s} \right)^{\sigma_s}, \tag{31}$$

where P^{va}_s is the composite value-added price: $P^{va}_s = \prod_f [(1+t_{fs})PF_f]^{\alpha^s_f} [(1+t^{res}_s)PF^g_{res}]^{\alpha^s_{res}}$. Note that PF^g_{res} is the price of sector-specific factor associated with natural resource and land. The market clearance condition for this natural resource and land factor input is given by:

$$\overline{S}^s_{res} = \alpha^s_{res} \beta^s_{vas} Y^s \left(\frac{P^{va}_s}{(1+t^{res}_s)PF_{res}} \right) \left(\frac{P^{vas}_s}{P^{va}_s} \right)^{\sigma_{vas}} \left(\frac{PY^s}{P^{vas}_s} \right)^{\sigma_s}, \tag{32}$$

In addition, we have the market for the specific factor used in the IRTS sectors. Denoting the regional endowments of the specific factors $\overline{SF}^g_r$ $\forall g \in (I \cup J)$, we have:

$$\overline{SF}^g_r = \theta^g_{Zr} Z^g_r \left(\frac{PMC^g_r}{PZ^g_r} \right)^{\epsilon^g_r} \quad \forall g \in (I \cup J). \tag{33}$$

Real investment equals real savings by households:

$$INV = \overline{sav}. \tag{34}$$

Real government purchases equal the nominal government budget scaled by the government price index:

$$PUB = \frac{GOVT}{PG}. \tag{35}$$

Household utility (U) equals nominal income across households scaled by the true-cost-of-living index. That is, we represent an aggregate activity U, which supplies *utils* to the households. For the representative agent of household type h denote nominal income

RA. The market clearance condition for *utils* is thus

$$U = \frac{RA}{PC}. \tag{36}$$

The final market clearance condition reconciles the balance of payments. The supply of foreign exchange includes its generation in the export activities and net borrowing from the rest of the world (net capital account surpluses). The real capital account surplus is held fixed at the exogenous benchmark observation, denoted $\overline{ftrn}$. Foreign exchange is demanded for direct import purchases as well as the payments to foreign agents for their contribution to production.

$$\begin{aligned} \sum_{r \neq D} \sum_{g} EX_r^g + \overline{ftrn} &= \sum_{r \neq D} \sum_{g} IM_r^g \\ &+ \sum_{r \neq D} \sum_{i} \theta_{Mr}^i Z_r^i \left(\frac{PMC_r^i}{(1 + t_i^{m,int}) PFX} \right)^{\epsilon_r^i} \\ &+ \frac{FE}{PFX}, \end{aligned} \tag{37}$$

where FE equals the nominal claims that the foreign entrepreneurs have on specific factor rents in the Dixit-Stiglitz manufacturing sectors.

3 Income Balance Conditions

The representative agent (household) earns income from factor endowments, but disposable income nets out savings and a direct tax transfer to the government. Real savings is held fixed (by the coefficient $\overline{sav}_h$). We also hold the real level of government spending fixed, but this requires an adjustment in direct taxes on households. Removal of tariffs, for example, impact the government budget and the shortfall is made up for by an endogenous increase in the direct taxes on households. We use the auxiliary variable T to scale the direct taxes appropriately. In addition, the household is assumed to hold any benchmark net international capital flows. The household's budget is given by

$$
\begin{aligned}
RA &= \sum_f PF_f \overline{S}_f \\
&= \sum_s PF^s_{res} \overline{S}^s_{res} \\
&+ \sum_g PZ^g_{UKR} \overline{SF}^g_{UKR} \\
&- \overline{sav} PINV \\
&- \overline{dtax} PG \times T \\
&+ \overline{ftrn} PFX
\end{aligned} \tag{38}
$$

The government budget is given by net direct and indirect taxes on domestic and international transactions. The full nominal government budget is

$$
\begin{aligned}
GOVT &= \overline{dtax}_h PG \times T \\
&+ \sum_g t^{cons}_g PA^g \mu^g_C U \frac{PC}{(1+t^{cons}_g)PA^g} \\
&+ \sum_g t^{inv}_g PA^g \mu^g_{INV} INV \\
&+ \sum_g t^{gov}_g PA^g \mu^g_G PUB \\
&+ \sum_i t^{d,int}_i PD_i \left(\theta^i_{DD} Z^i_D \left(\frac{PMC^i_D}{PD^i} \right)^{\epsilon^i_D} + \sum_{r \neq D} \theta^i_{Dr} Z^i_r \left(\frac{PMC^i_r}{PD^i} \right)^{\epsilon^i_r} \right) \\
&+ \sum_j t^{d,int}_j PD_j \theta^j_{DD} Z^j_D \left(\frac{PMC^j_D}{PD^j} \right)^{\epsilon^j_D} \\
&+ \sum_k t^{d,int}_k PD_k \phi^k_D A^k \left(\frac{PA^k}{PD^k} \right)^{\sigma^k_{DM}} \\
&+ \sum_s \sum_f t_{fs} PF_f \alpha^s_f \beta^s_{vas} Y^s \left(\frac{P^{va}_s}{(1+t_{fs})PF_f} \right) \left(\frac{P^{vas}_s}{P^{va}_s} \right)^{\sigma_{vas}} \left(\frac{PY^s}{P^{vas}_s} \right)^{\sigma_s} \\
&+ \sum_s t^{res}_s PF_{res} \alpha^s_{res} \beta^s_{vas} Y^s \left(\frac{P^{va}_s}{(1+t^{res}_s)PF_{res}} \right) \left(\frac{P^{vas}_s}{P^{va}_s} \right)^{\sigma_{vas}} \left(\frac{PY^s}{P^{vas}_s} \right)^{\sigma_s}
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum_{r \neq D} \sum_{g} t_g^{m,int} PM_r^g IM_r^g \\
&+ \sum_{r \neq D} \sum_{i} t_i^{m,int} PM_r^i \theta_{Mr}^i Z_r^i \left(\frac{PMC_r^i}{(1 + t_i^{m,int}) PM_r^i} \right)^{\epsilon_r^i} \\
&+ \sum_{r \neq D} \sum_{g} t_g^{exp} PX_r^g EX_r^g
\end{aligned} \tag{39}
$$

Again, the index T is adjusted endogenously to hold the real level of public spending fixed. In addition to the household and government agents we need an agent representing the foreign entrepreneurs who own the specific factors associated with cross-border Dixit-Stiglitz traded goods. The foreign entrepreneur's nominal income is FE, which is spent on foreign exchange:

$$
FE = \sum_{r \neq D} \sum_{g} PZ_r^g \overline{SF}_r^g \tag{40}
$$

4 Auxiliary Condition

In addition to the three sets of standard conditions presented above, we need to close the model with an auxiliary condition such that the real size of the government is held fixed. To do this we need to determine the index which scales direct taxes on households. Associated with the variable T is the following condition:

$$
PUB = \overline{pub}. \tag{41}
$$

References

Balistreri, Edward J., Zoryana Olekseyuk, and David G. Tarr (2017) 'Privatisation and the unusual case of belarusian accession to the wto.' *The World Economy* 40(12), 2564–2591